

注意：考試開始鈴響前，不得翻閱試題，
並不得書寫、畫記、作答。

國立清華大學 112學年度學士後醫學系單招試題

系所班組別：學士後醫學系
智慧資訊科技組

科目代碼：0104

考試科目：資訊科學

—作答注意事項—

1. 請核對答案卡上之准考證號、科目名稱是否正確。
2. 作答中如有發現試題印刷不清，得舉手請監試人員處理，但不得要求解釋題意。
3. 答案卡限用 2B 鉛筆畫記；如畫記不清（含未依範例畫記）致光學閱讀機無法辨識答案者，其後果一律由考生自行負責。
4. 其他應考規則、違規處理及扣分方式，請自行詳閱簡章附錄上「**國立清華大學試場規則及違規處理辦法**」，無法因本試題封面作答注意事項中未列明而稱未知悉。

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【單選題】

**** 答錯一題倒扣 1.25 分 **** 未作答, 不給分亦不扣分。

1. (5%) Let P_2 denote the collection of all real polynomials of degree less than or equal to 2. Define the linear transformation $L: P_2 \rightarrow P_2$ by $L(p(x)) = p'(x) - p(1)x^2 + p(-1)$ for any polynomial $p(x)$ in P_2 . What is $\det(L)$?

(A) -3

(B) -2

(C) 0

(D) 1

(E) 2

2. (5%) Let $\mathbb{R}^{2 \times 2}$ denote the collection of all real 2×2 matrices and P denote the collection of all real polynomials. Which two of the following functions are linear operators?

$$T_1: \mathbb{R} \rightarrow \mathbb{R}, T_1(x) = 2x - 3$$

$$T_2: \mathbb{R}^{2 \times 2} \rightarrow \mathbb{R}, T_2(A) = \det(A)$$

$$T_3: \mathbb{R}^{2 \times 2} \rightarrow \mathbb{R}^{2 \times 2}, T_3(A) = A^T - A$$

$$T_4: P \rightarrow P, T_4(f(x)) = f'(x) + f(0)x \text{ for any polynomial } f(x)$$

(A) T_1, T_2

(B) T_1, T_3

(C) T_2, T_3

(D) T_2, T_4

(E) T_3, T_4

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3. (5%) Let $\beta = \{(1,1), (2,1)\}$ be an ordered basis of \mathbb{R}^2 . Suppose P is the orthogonal projection onto $\text{span}\{(0,1)\}$. What is the matrix representation $[P]_\beta$ of P in β ?

(A) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix}$

(E) $\begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}$

4. (5%) Which statement below is correct?

(A) Any determinant is a linear transform

(B) Similar matrices always have the same eigenvalues

(C) The zero vector space has no basis

(D) Every system of n linear equations in n unknowns can be solved by Cramer's rule

(E) Only square matrices have a conjugate-transpose

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5. (5%) Which of the following number β can make the matrix B positive definite?

$$B = \begin{pmatrix} \beta & 1 & 1 \\ 1 & \beta & 1 \\ 1 & 1 & \beta \end{pmatrix}$$

- (A) -1
- (B) 0
- (C) 1
- (D) 2
- (E) i

6. (5%) Determine

$$\lim_{m \rightarrow \infty} \begin{pmatrix} -\frac{1}{2} - 2i & 4i & \frac{1}{2} + 5i \\ 1 + 2i & -3i & -1 - 4i \\ -1 - 2i & 4i & 1 + 5i \end{pmatrix}^m$$

- (A) 1
- (B) -1
- (C) 0
- (D) i
- (E) Does not exist

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7. (5%) Consider a regular transition matrix for a three-state Markov chain, with the initial probability vector $\begin{pmatrix} 0.3 \\ 0.3 \\ 0.4 \end{pmatrix}$,

$$\begin{pmatrix} 0.6 & 0.1 & 0.1 \\ 0.1 & 0.9 & 0.2 \\ 0.3 & 0 & 0.7 \end{pmatrix}$$

Find the proportions of objects eventually.

(A) $\begin{pmatrix} 0.3 \\ 0.3 \\ 0.4 \end{pmatrix}$

(B) $\begin{pmatrix} 0.2 \\ 0.4 \\ 0.4 \end{pmatrix}$

(C) $\begin{pmatrix} 0.2 \\ 0.5 \\ 0.3 \end{pmatrix}$

(D) $\begin{pmatrix} 0.2 \\ 0.6 \\ 0.2 \end{pmatrix}$

(E) $\begin{pmatrix} 0.3 \\ 0.5 \\ 0.2 \end{pmatrix}$

8. (5%) Which of the following vectors are linearly independent in the vector space R^3 ?

(A) $[1, 1, 3]^T, [0, 2, 1]^T, [2, 0, 5]^T$

(B) $[1, 1, 0]^T, [0, 1, 1]^T, [2, 1, -1]^T$

(C) $[4, 1, -3]^T, [1, 0, -1]^T, [1, 2, 1]^T$

(D) $[1, -1, 2]^T, [1, 1, 7]^T, [2, 0, -1]^T$

(E) $[1, -1, 2]^T, [1, 0, 7]^T, [2, -3, -1]^T$

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9. (5%) Consider the system of linear equations:

$$x + 2y + 4z = 8$$

$$x + y + 3z = -2$$

$$x - 3y - z = 0$$

Which of the following is **NOT** the least square solution of this system?

- (A) $[2, 1, 0]^T$
- (B) $[-4, -2, 3]^T$
- (C) $[4, 2, -1]^T$
- (D) $[-2, -1, 1]^T$
- (E) $[-2, -1, 2]^T$

10. (5%) Which of the following matrices is diagonalizable?

(A) $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$

(B) $\begin{bmatrix} -2 & 0 & 1 \\ -6 & -2 & 0 \\ 19 & 5 & -4 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 0 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 19 & -9 & -6 \\ 25 & -11 & -9 \\ 17 & -9 & -4 \end{bmatrix}$

(E) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ -3 & 5 & 2 \end{bmatrix}$

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11. (5%) Suppose that the characteristic polynomial of a matrix A is

$$p(\lambda) = \lambda(\lambda - 1)(\lambda - 2)^2(\lambda + 3)^3.$$

Which of the following statements about the matrix A is correct?

- (A) The size of A is 4×4
- (B) A is invertible
- (C) $\det(A) = -128$
- (D) $(2I - A)\mathbf{x} = \mathbf{0}$ has nontrivial solutions
- (E) A has nullity 0

12. (5%) Let A be an n by n matrix. Which of the following statement is **NOT** identical to others?

- (A) $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution
- (B) The rank of A is less than n
- (C) The dimension of A 's null space is positive
- (D) A is not diagonalizable
- (E) The singular values of A are not all positive

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13. (5%) Let v_1, \dots, v_n be vectors in a vector space V for $n > 0$. Which statement is **NOT** correct?

- (A) If the dimension of V is n , and v_1, \dots, v_n span V , then v_1, \dots, v_n are linearly independent
- (B) If the dimension of V is m , and $m > n$, then v_1, \dots, v_n can be extended to form a basis for V
- (C) If the dimension of V is m , and $m < n$, then v_1, \dots, v_n are linearly dependent
- (D) If both $\{v_1, \dots, v_n\}$ and $\{u_1, \dots, u_m\}$ are bases for a vector space V , then $n = m$
- (E) A vector v in $\text{Span}(v_1, \dots, v_n)$ can be written uniquely as a linear combination of v_1, \dots, v_n if and only if v_1, \dots, v_n are linearly independent

14. (5%) Which one is **NOT** an eigenvalue of $A = \begin{bmatrix} 4 & 0 & 7 & 3 \\ 1 & 3 & 1 & 2 \\ 0 & 0 & 2 & 0 \\ 1 & 0 & 5 & 2 \end{bmatrix}$?

- (A) 1
- (B) 2
- (C) 3
- (D) 4
- (E) 5

15. (5%) Let $A = \begin{bmatrix} 4 & 6 & -2 \\ -1 & -1 & 1 \\ 0 & 0 & \alpha \end{bmatrix}$. Which value of α can make A defective?

- (A) No value of α can make A defective
- (B) -2
- (C) 2
- (D) 0
- (E) 1

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16. (5%) Kim is at a train station, waiting to make a phone call. Two public telephone booths, next to each other, are occupied by two callers, and 11 persons are waiting in a single line ahead of Kim to call. If the duration of each telephone call is an independent exponential random variable with $\lambda = 1/3$, what is the variance of Kim's waiting time? Kim's waiting time is defined as the time interval from now to the moment that a phone is available for Kim to make his call.
- (A) 18
(B) 27
(C) 99/4
(D) 99
(E) 108
17. (5%) Let X and Y be independent random points from the interval $(-1, 1)$. Find $E(\max(X, Y))$.
- (A) 2/3
(B) 1/2
(C) 1/3
(D) 1/4
(E) 0
18. (5%) A stick of length 1 is broken into two pieces at a random point. Find the correlation coefficient and the covariance of these pieces.
- (A) 1/2
(B) -1/2
(C) 1/12
(D) -1/12
(E) 0

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19. (5%) A traffic light can be in one of the three states: Green (G), Red (R), and Yellow (Y). The light changes in a random fashion. At any one time, the light can be in only one state. The experiment consists of observing the state of the light.

Let a random variable $X(\cdot)$ be defined as follows: $X(G) = -1$, $X(R) = 0$, $X(Y) = \pi$; assume that $P(G) = P(Y) = 0.5 P(R)$, what is $P(X \leq 3)$?

- (A) $1/4$
- (B) $1/2$
- (C) $3/4$
- (D) 1
- (E) 0

20. (5%) Let X_1, X_2, \dots, X_n be n identical and independently distributed exponential random variables with $f_{X_i}(x) = e^{-x}u(x)$, $u(x) = 1$ for $x \geq 0$, 0 otherwise. The maximum value for the probability density function, $f_{Z_n}(z)$, of the random variable $Z_n = \max(X_1, X_2, \dots, X_n)$ occurs at $z = ?$

- (A) $\ln(n)$
- (B) $\ln(-1)$
- (C) $\ln\left(\frac{1}{n}\right)$
- (D) $(1 - e^{-1})^n$
- (E) $n(1 - e^{-1})^n$

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21. (5%) Y_1, Y_2, \dots are independent and identically distributed random variables; the moment generating function of each of them is $M(t) = \left(\frac{1}{3} + \frac{2}{3}e^t\right)^{10}$. Let $X = \sum_{n=1}^3 Y_n$. Find the probability mass function of $X, p_X(x)$.

(A) $p_X(x) = e^{-10} \frac{10^x}{x!}, x = 0, 1, 2, \dots$

(B) $p_X(x) = e^{-20} \frac{20^x}{x!}, x = 0, 1, 2, \dots$

(C) $p_X(x) = \binom{20}{x} \left(\frac{2}{3}\right)^x \left(\frac{1}{3}\right)^{20-x}, x = 0, 1, \dots, 20$

(D) $p_X(x) = \binom{30}{x} \left(\frac{2}{3}\right)^x \left(\frac{1}{3}\right)^{30-x}, x = 0, 1, \dots, 30$

(E) $p_X(x) = \binom{x-1}{20-1} p^{20} (1-p)^{x-20}, x = 20, 21, 22, \dots$

22. (5%) A newly married couple decides to continue having children until they have one of each sex. If the probability of a child being a boy is $1/4$ and the probability of a child being a girl is $3/4$, how many children should this couple expect?

(A) 4

(B) $\frac{17}{4}$

(C) $\frac{13}{3}$

(D) $\frac{9}{2}$

(E) $\frac{16}{3}$

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23. (5%) The two random variables, X and Y , are independent and exponentially distributed, each with mean $1/\lambda$. Let $Z = \frac{X}{X+2Y}$. Find $F_Z(z)$, which is the cumulative distribution function of Z .

(A) $F_Z(z) = \frac{2}{\pi} \int_0^z \frac{1}{1+x^2} dx = \begin{cases} \frac{2}{\pi} \cdot \tan^{-1} z, & z \geq 0 \\ 0, & \text{otherwise} \end{cases}$

(B) $F_Z(z) = \begin{cases} \frac{3z}{3z+1}, & z \geq 0 \\ 0, & \text{otherwise} \end{cases}$

(C) $F_Z(z) = \begin{cases} \frac{z}{z+1}, & z \geq 0 \\ 0, & \text{otherwise} \end{cases}$

(D) $F_Z(z) = \begin{cases} 0, & z < 0 \\ z, & 0 \leq z \leq 1 \\ 1, & z > 1 \end{cases}$

(E) $F_Z(z) = \begin{cases} 0, & z < 0 \\ \frac{2z}{1+z}, & 0 \leq z \leq 1 \\ 1, & z > 1 \end{cases}$

24. (5%) We have some earthquake data that occurred in a region over years, summarized in the following table:

Magnitude	$M < 3$	$3 \leq M$ < 4	$4 \leq M$ < 5	$5 \leq M$ < 6	$6 \leq M$
Frequency (times/per year)	150	40	15	6	2

Assume that earthquakes are independent rare events and their occurrences are homogeneous over time. Under these assumptions, the number of earthquake events follows Poisson distribution. Estimate the probability of having earthquakes with magnitude $M \geq 5$ in the next month.

(A) 3.8%

(B) 21.3%

(C) 48.7%

(D) 66.7%

(E) unable to evaluate

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25. (5%) Which of the following statements are correct?

- (1) Statistical independence implies no correlation
- (2) No correlation implies statistical independence
- (3) Statistical independence and no correlation are equivalent
- (4) In a simple linear regression problem, a large value of the least square estimate for the slope coefficient implies a strong association between the response and the explanatory variables

- (A) (1)
- (B) (2)
- (C) (3)
- (D) (1)(4)
- (E) (1)(2)(3)(4)

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26. (5%) Hospital A is experimenting with an AI-assisted system for judging the status (+ or -) of a certain disease. A pilot study is conducted to compare the results assessed by the AI system and a trained doctor. 50 patients are recruited, and the results are reported in the following table:

Patient	Disease Status	
	AI-assisted System	Trained Doctor
1	—	—
2	—	—
3	+	—
4	+	+
5	—	—
6	+	+
7	+	—
⋮	⋮	⋮
50	—	—

The goal of the study is to verify if the assessment from the AI system is consistent with that from a trained doctor. Which of the following statistical test can be used to answer the question?

- (A) 5 independent 2 sample t-test
- (B) paired 2 sample t-test
- (C) chi-squared test
- (D) signed-rank test
- (E) F test

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27. (5%) Based on the playing history of Team A and Team B, Team A has a 60% chance of winning basketball in one game. Assume that the outcomes of their games are independent. Which of the following statements is correct?

- (A) Team A has a better chance of winning in 2-out-of-3 gameplay (三戰兩勝) rather than in 3-out-of-5 gameplay (五戰三勝)
- (B) Team A has a more than 70% chance of winning in 2-out-of-3 gameplay
- (C) For 2-out-of-3 gameplay, the chance of playing the third game is more than 40%
- (D) Given that the third game takes place in 2-out-of-3 gameplay, Team B has a less than 40% chance of winning the game
- (E) None of the above

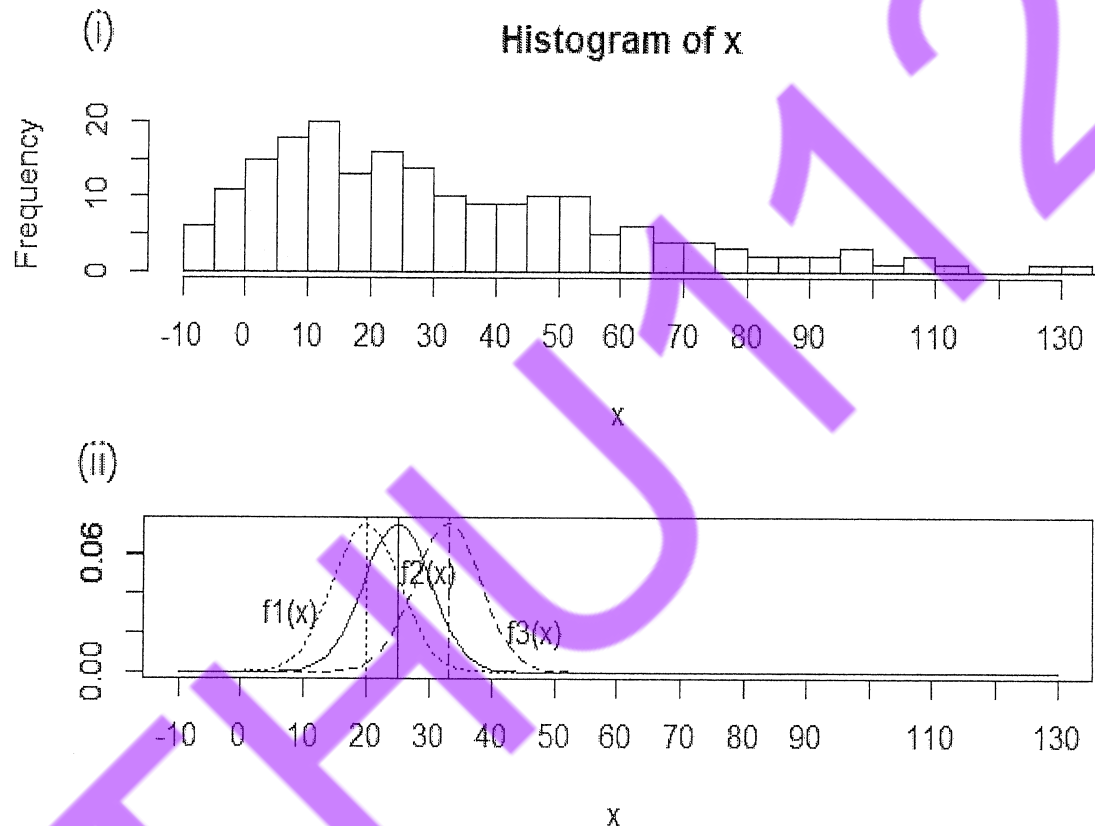
28. (5%) From the general knowledge, the population occurrence rate of asthma is 0.1 for children who visit hospitals. A pediatrician moved his clinic to a new place and he found 9 children with asthma among 50 appointments in his first day at the new place. If the 50 appointments are considered as Bernoulli random variables with 0 and 1 outcomes, the total number of asthma children, indicated as X , should follow a Binomial distribution with probability 0.1 under the general condition. He then makes the following guesses. Which one is a correct inference for the occurrence rate at the new place?

- (A) If the occurrence rate holds the same as other places, the standard deviation of each Bernoulli random variable in this problem is $\sqrt{0.1 \times (1 - 0.1)} = 0.3$. The standard deviation of X should be $50 \times 0.3 = 15$
- (B) Since $9/50 = 0.18$ is greater than 0.1, there must be an increase of the occurrence rate
- (C) If the occurrence rate is greater than 0.1, we should also see the proportion of asthma children to be greater than 0.1 in the next day
- (D) Since $P(X=9) = 0.033$ is smaller than 0.05, we will decide this as an unusual situation, and the evidence supports an increased occurrence rate. The threshold 0.05 can be applied when the number of appointments is 70 in a day
- (E) Since $P(X \geq 9) = 0.058$ is greater than 0.05, we do not have to worry too much about the situation, and it might be a random fluctuation from day to day. The threshold 0.05 can be applied when the number of appointments is 70 in a day

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29. (5%) A histogram of a dataset X is plotted in Figure (i). The 1st quartile, median and the 3rd quartile of the data are 10, 25, and 50. Which of the following statements is correct?



- (A) The truncated mean of the central 50% data should be $(10+50)/2=30$
- (B) If another variable Y is positively correlated with X , the distribution of Y must be skew to the right as the distribution of X
- (C) The mode is greater than 25
- (D) Since it is a skewed distribution, we can model the data with a χ^2 distribution
- (E) Suppose we randomly draw 30 samples with replacement from this dataset and take average to get \bar{X} . The distribution of \bar{X} is more likely to be the function $f3(x)$ in Figure (ii) rather than the function $f1(x)$ or $f2(x)$

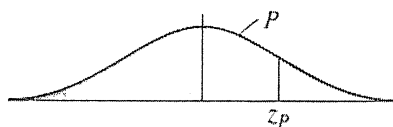
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30. (5%) Which of the following is **NOT** a reason for conducting the paired-sample T-test?
- (A) The sample size is too small
 - (B) The treatments can be compared with homogeneous subjects
 - (C) The variation of measurements is too large compared to the variation related to the treatment
 - (D) The data collected are paired with positive correlations
 - (E) The subjects differ a lot in terms of their age, gender and ethnic groups

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z is the standard normal variable. The value of P for $-z_p$ equals 1 minus the value of P for $+z_p$; for example, the P for -1.62 equals $1 - .9474 = .0526$.

[illegible]